JOURNAL OF
HYDRO-METEOROLOGY

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*Research Article*

# Water level forecasting at Hanoi station using transformer-based AI models

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Received: 05 October 2024; Accepted: 10 December 2024; Published: 25 March 2025

**Abstract:** Flood forecasting is the main task to mitigate the damage caused by flooding in the Red River, Vietnam. Many reservoirs have been operating in the Red River to regulate flood. This research aims at developing a method for rapidly forecasting Hanoi's water levels under various reservoir operation scenarios and river system conditions that will facilitate the assessment of multiple reservoir operation scenarios to provide effective and reliable real-time operational advice. A deep learning model based on the Transformer architecture was used to forecast the 24-hour lead time at Hanoi station. The dataset was divided into three subsets (Training set from 2015 to 2022, validation set in 2023 and Test set in 2024). The results showed that the Mean Absolute Error (MAE) was within an acceptable range, with MAE values of 24.1 cm, 26.1 cm, and 30.7 cm for the training, validation, and testing phases, respectively. The model demonstrated a significant ability to capture historical patterns and achieve high accuracy on the validation dataset, emphasizing the effectiveness of the Transformer architecture in forecasting water levels under normal conditions. Hydraulic models can be used to simulate additional data to improve the quality of flood forecasts for these extreme cases.

**Keywords:** Flood Forecasting; Artificial Intelligence; Transformers; Red River.

## 1. Introduction

Flood forecasting is one of the most effective non-structural measures to mitigate the impacts of flooding. The Red River plays a crucial role in the socio-economic development of the Northern Delta region, home to the capital city of Hanoi, a cradle/center of Vietnam's culture, politics and education. To ensure the safety of Hanoi and the Red River Delta, a system of dikes has been constructed along the river, along with large reservoirs upstream of the Da and Lo Rivers to regulate floods.

Several studies reveal that the traditional flood forecasting approach in the Red River basin at Hanoi has been applied using hydrological-hydraulic models with reservoir operation models [1–4]. Hydrological models use forecasted precipitation data to generate forecasted runoff for sub-basins, which is then used to estimate flows at control points in the upstream basin. These control points are typically hydrological stations or reservoirs. The runoff is then routed to the main reservoirs using hydrological routing models. Reservoir operation models will calculate the released flow based on the reservoir operation scenarios. Finally, hydraulic models will simulate the downstream flow, accounting for tidal levels and interactions with adjacent river branches.

Under the Red River inter-reservoir operation regulation 740/QĐ-TTg 2019 [5], seven reservoirs on the mainstream participate in flood regulation and control: Lai Chau, Son La, Hoa Binh, Huoi Quang, Ban Chat, Thac Ba, and Tuyen Quang. Among these, the operational flows from the two main downstream reservoirs, Hoa Binh and Tuyen Quang, directly influence water levels in Hanoi. To provide real-time operational advice for this reservoir system, ensuring the safety of the structures and downstream areas, the outflows of reservoirs under different scenarios are linked to hydraulic models to simulate downstream flow. With this approach, it requires significant time for real-time forecasting, especially when considering multiple operational scenarios.

Consequently, developing a method for rapid water level forecasting in Hanoi under various reservoir operation scenarios and river system conditions will facilitate the assessment of multiple reservoir operation scenarios, providing effective and reliable real-time operational advice.

Artificial intelligence (AI) technology has been advancing rapidly and is being successfully utilized in various practical fields. Several studies have employed AI-based approaches for flood forecasting [6–11]. The results of these studies show the potential applications of AI in flood management. In Vietnam, AI has been applied in flood forecasting for decades. Most of the studies used the popular AI algorithms [12–14] such as artificial neural networks, long-short-term memory (LSTM)… These models have good accuracy but still have some limitations, for example, ANN has difficulty in dealing with time series data [15], LSTM requires a large quantity of data [16] or is prone to overfitting [17].

The Transformer is a neural network architecture designed to map input sequences to output sequences, first introduced in [18]. It offers significant advantages, including the self-attention mechanism, which enables parallel computation and improves performance on long time-series tasks due to its extended memory capacity [19]. Researchers have applied successfully the Transformer model in flood forecasting in several basins in the world [20–22].

Therefore, the study objective is to use the Transformer model to forecast water levels in Hanoi with a 24-hour lead time.

## 2. Methods and data

### *2.1. Study area*

The Red - Thai Binh River basin, covering an area of 169,000 km², is the second-largest river basin in Vietnam after the Mekong River basin (Figure 1). In the Red River basin, many reservoir systems are operating to regulate flood control for downstream areas as well as provide water for different water requirements. Hanoi is located downstream in the Red River basin, below the confluence of its three main tributaries: the Da River, the Thao River, and the Lo River. Water levels in Hanoi are affected by upstream flows as well as the water levels of tributaries in the downstream area.

### *2.2. Methodology*

In this section, the research introduces a deep learning model based on the Transformer architecture [23], designed to predict water levels in Hanoi for the next 48 hours (equivalent to the upcoming two days). The overview architecture of the proposed model is shown in Figure 2. This consists of several key components: a Time-Embedding block, two Encoder blocks, one for information from the hydrological stations and the other for data from the reservoirs, and a Decoder block to decode the information and predict the water levels at the upcoming time steps.

The time-embedding block is used to encode the time information and inject temporal patterns into the model. It is typically done using sine and cosine functions, which help the model capture periodicity, such as daily or yearly cycles. Given a time step t, the Time-Embedding for that time is calculated as follows:

$$\mathrm{TE}(t,2i)=\sin\left(\frac{t}{f^{(2j/d)}}\right);\mathrm{TE}(t,2i+1)=\cos\left(\frac{t}{f^{(2j/d)}}\right) \quad (1)$$

where i is the dimension of the embedding (ranging from 0 to $\frac{d}{2}$) and d is the total number of embedding dimensions. These embeddings allow the model to better understand and process temporal information, such as the day of the year and season, making it suitable for time-series prediction tasks such as water level forecasting.

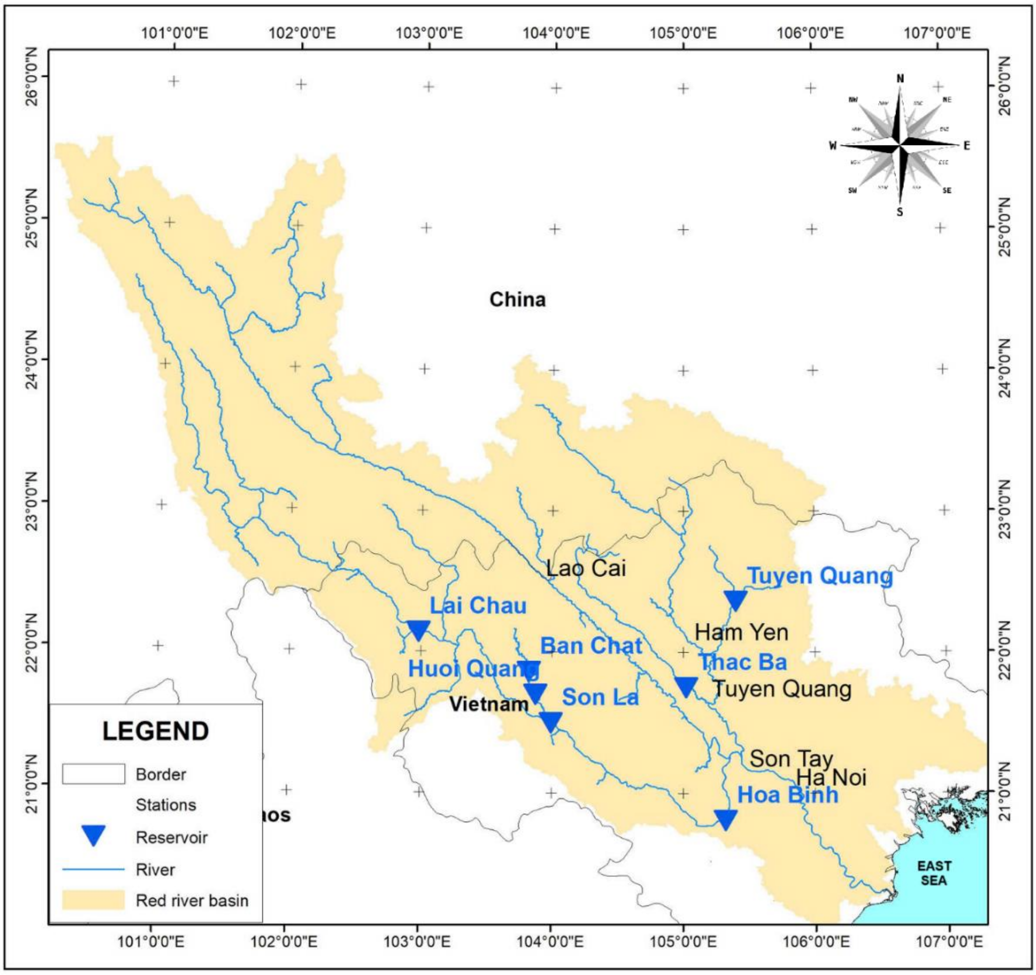


**Figure 1.** Map of the Red River Basin with main reservoirs.

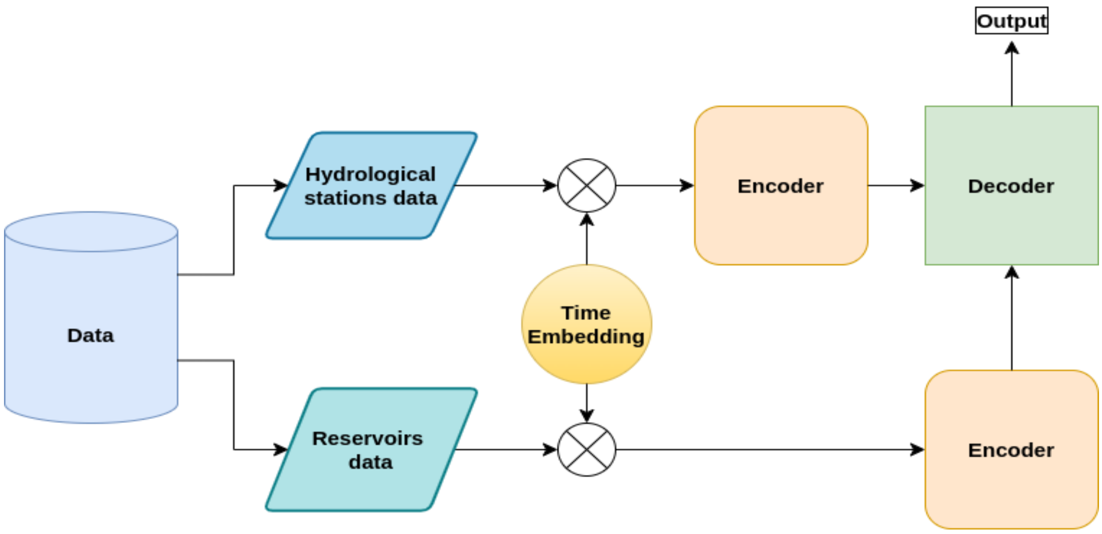


**Figure 2.** Pipeline of the Model for Predicting Water Levels in Hanoi.

The Encoder block is designed separately for each type of input data from the hydrological stations and the reservoirs because the length of the data varies between them. The architecture of the Encoders is designed with a combination of Positional Encoding [24] and stacked Transformer Encoder blocks. Unlike Time-Embedding, which encodes time information within a window of length 36, Positional Encoding encodes the temporal information specific to each time step in the sequence, allowing the model to learn relationships within a short time frame, such as the past week, to capture the temporal dependencies relevant for accurate forecasting. Following this, each Transformer Encoder block is designed with feed-forward networks (FFN) and multi-head self-attention layers (MHSA) to enrich the information from each hydrological station and reservoir at every time step. MHSA (Multi-Head Self-Attention) revolves around the attention mechanism with multiple heads, allowing the model to capture different aspects of relationships between time steps in the data. Each time step is processed with query, key, and value vectors to determine the attention weights between time steps. This process can be defined by the following formula:

$$\mathrm{Attention}\left(Q,K,V\right)=\mathrm{soft\,max}\left(\frac{QK^{T}}{\sqrt{d_k}}\right)V \tag{2}$$

This enables the model to focus on key time steps, even if they are not adjacent, and learn long-range dependencies. FFN layers are applied after the attention mechanism to process and transform the information, adding the ability to capture non-linear relationships in the data. The FFN layers consist of two fully connected layers with a ReLU activation function, allowing the model to learn complex patterns in the time series data.

The Decoder is responsible for synthesizing the information encoded from the hydrological stations and reservoirs through the two preceding Encoders. It is also designed with stacked Transformer Decoder blocks, which consist of feed-forward networks and cross-attention mechanisms. Cross-attention [18] allows the model to focus on the relationship between the hydrological stations and reservoirs, enabling it to effectively combine information from both sources. This helps the model learn how the data from two sources interact to make accurate predictions for future water levels. Additionally, an advanced mechanism, masked cross-attention [25], is employed to ensure that the model does not have access to future data during training. This mechanism prevents data leakage and ensures that the model learns to predict future water levels based only on the past and current information, maintaining the temporal integrity of the forecasting process.

At the output stage, the Decoder produces the predicted water levels for future time steps, serving as the model's forecast for the upcoming water levels in Ha Noi. To finalize the predictions, a post-processing step is conducted to convert the normalized outputs back to their original scale by reversing the normalization applied during preprocessing. This ensures the predictions are accurate, interpretable, and ready for practical use in water level forecasting.

### *2.3. Data collection*

#### 2.3.1. Hydrological Data

Observed flow data from 2014 and earlier will not be used to avoid the impact of riverbed changes on water levels. Therefore, water level data from hydrological and primary tidal stations in the Red River system have been collected from 2015 to the present.

In the Da River system, as the Hoa Binh Reservoir, located on the mainstream, plays an important role in regulating the flow, only the outflow data from the Hoa Binh Reservoir will represent flow information for the Da River branch. For the Thao River, data from the Lao Cai and Yen Bai stations provide information for this branch. In the Lo-Gam River system, outflow data from the Tuyen Quang and Thac Ba reservoirs are utilized, along with water level data from stations at Ha Giang, Bac Me, Vinh Tuy, Ham Yen, and Vu Quang. In the lower sections of the Red and Thai Binh Rivers, several water level data in hydrological stations are also used. To ensure consistency, water level and flow data are recorded at 6-hour intervals for all days from 2015 to September 2024.

### 2.3.2. Data Pre-Processing

The problem of water level forecasting for the upcoming days is approached by using historical water level data collected from 19 hydrological stations and three reservoirs. The data consists of observations recorded over nearly a decade, ranging from January 1, 2015, to September 28, 2024. Measurements were recorded four times daily at 1:00 AM, 7:00 AM, 1:00 PM, and 7:00 PM, with water levels reported in centimetres relative to sea level.

Prior to using the data for subsequent model training steps, preprocessing techniques are implemented to create a standardized dataset appropriate for the model. Since the data was collected in real-world conditions, errors, and missing values over the nearly 10-year recording period are unavoidable. Consequently, the data were selected based on their availability, quality, and impact on the predictor.

After the preprocessing procedure, the data of 19 hydrological stations and three reservoirs were processed using interpolation techniques to fill in the gaps, ensuring that the dataset remains complete and continuous. This approach avoids disruptions in the analysis and model training process. Subsequently, the data was standardized using the standard normalization method, which scales all measured values to a common range, eliminating the impact of differing measurement units across locations. This ensures that no feature is disproportionately weighted during the model's learning process. Finally, the dataset was divided into three subsets: (1) Training set: Consists of data from January 1, 2015, to December 31, 2022; (2) Validation set: Contains data from January 1, 2023, to December 31, 2023; (3) Test set: Includes data from January 1, 2024, to September 28, 2024.

This partitioning ensures a clear separation of temporal phases, allowing the model to be trained on historical data while maintaining its ability to accurately predict unseen data. These preprocessing steps not only enhance the data quality but also lay a solid foundation for the entire model training and evaluation pipeline.

With each dataset and four measurements per day, a sliding window of 36 timesteps (equivalent to 9 days) is applied to the dataset, generating data points for the model. This window is shifted with a step size of four timesteps (equivalent to one day) across the datasets to create data points. Within each window, the first 28 timesteps (equivalent to a week) are considered past data, and the last 8 timesteps (equivalent to two days) are considered future data. A total of eight data points is generated to predict the upcoming eight-time steps, corresponding to the next two days. Specifically, at time $t$, the data point used to predict the water level at future time $t + \Delta t$ has the following structure:

Input: this combined water level data from 12 stations up to time $t$ and released discharge data from three reservoirs up to time $t + \Delta t$.

- Data from stations consists only of passive data, which includes the first 28 measurements (equivalent to a week) for each hydrological station within the window. In addition, each measurement includes its corresponding time step within the year and the specific month it was recorded.

- Data from reservoirs consists of both passive and future data. This includes the first 28 measurements from the past (equivalent to a week) and $\Delta t$ from the future for each reservoir within the window. Along with this, the corresponding time steps relative to the year and the specific month information are provided for each measurement.

Output: the target value is the water levels in Ha Noi and corresponding timestep in year, which need to be predicted for future time $t + \Delta t$.

## 3. Results

### *3.1. Experimental Setup*

For the experiments in this research, we developed a deep learning model based on the Transformer architecture to forecast water levels in Ha Noi. To embed time and sequence information, we used different frequencies for specific embeddings: a frequency of 5000 for month embeddings, 10000 for time step embeddings within the year, and 1000 for positional encoding. This approach ensures the model effectively captures various temporal patterns and relationships in the data. For the stacking layers in the Encoder and Decoder, we selected two layers for each block, with each Multi-Head Self-Attention (MHSA) layer using four heads and a dimension of 64 for the model. During training, dropout was applied to prevent overfitting, and input data was augmented by masking portions of the data to help the model generalize better to unseen scenarios and avoid overfitting, ensuring robust and accurate predictions.

In addition, the loss function used was mean squared error (MSE), while mean absolute error (MAE) was used as the evaluation metric. MSE is suitable for this problem as it penalizes large errors, prioritizing the minimization of critical prediction deviations, which is vital in water level forecasting. Meanwhile, MAE offers an intuitive measure of average error, less sensitive to outliers, ensuring a balanced and interpretable evaluation of model performance. There is also consideration given to the errors in extreme cases, ensuring the model remains robust under critical scenarios. This combination allows for effective training and practical evaluation relevant to real-world conditions.

### *3.2. Experimental Results*

The results of the experiments show that the Transformer-based model can effectively learn the relationships in the data between the hydrological stations and reservoirs, while also capturing the temporal dependencies to make accurate predictions of the water levels in Ha Noi. The results of the training, validation, and testing processes are measured by the MAE (Mean Absolute Error) metric to represent the prediction error and are presented in Table 1.

**Table 1.** Training Results of Transformer-Based AI Models

| Dataset | Trainset | Validation set | Test set |
|---|---|---|---|
| MAE (cm) | 24.1 | 26.1 | 30.7 |

The MAE metrics show that the model learns effectively on the training set and performs well on the validation set. On the training set, the mean deviation is 24.1 cm, while on the validation set, it is 26.1 cm, indicating that the error between the predicted and actual values is relatively low. This suggests that the model does not exhibit overfitting and is able to generalize well, understanding the characteristics of the data over time. However, the results on the test set are slightly higher (30.7 cm), but still promising. The model maintains good prediction capability, as the test set data corresponds to a year with more significant fluctuations due to the impact of natural disasters, such as Typhoon Yagi. Similar research using ANN and LSTM got the MAE around 10 cm but it is because of simulating instead of forecasting the water level at Hanoi, and daily data (smoother) instead of 6-hour data (strong variability) [14].

Water level forecasting in Hanoi is considered satisfactory if the forecast error is within 20 cm (the acceptance error is estimated following the circular 22/2019/TT-BTNMT). Therefore, with an error range of approximately 24-30 cm, the results of rapid water level forecasting in Hanoi using the Transformer-based AI method are acceptable, especially considering the challenges posed by extreme events like Typhoon Yagi. This suggests that the model performs well even in the presence of fluctuating and extreme conditions, making it suitable for real-world applications.

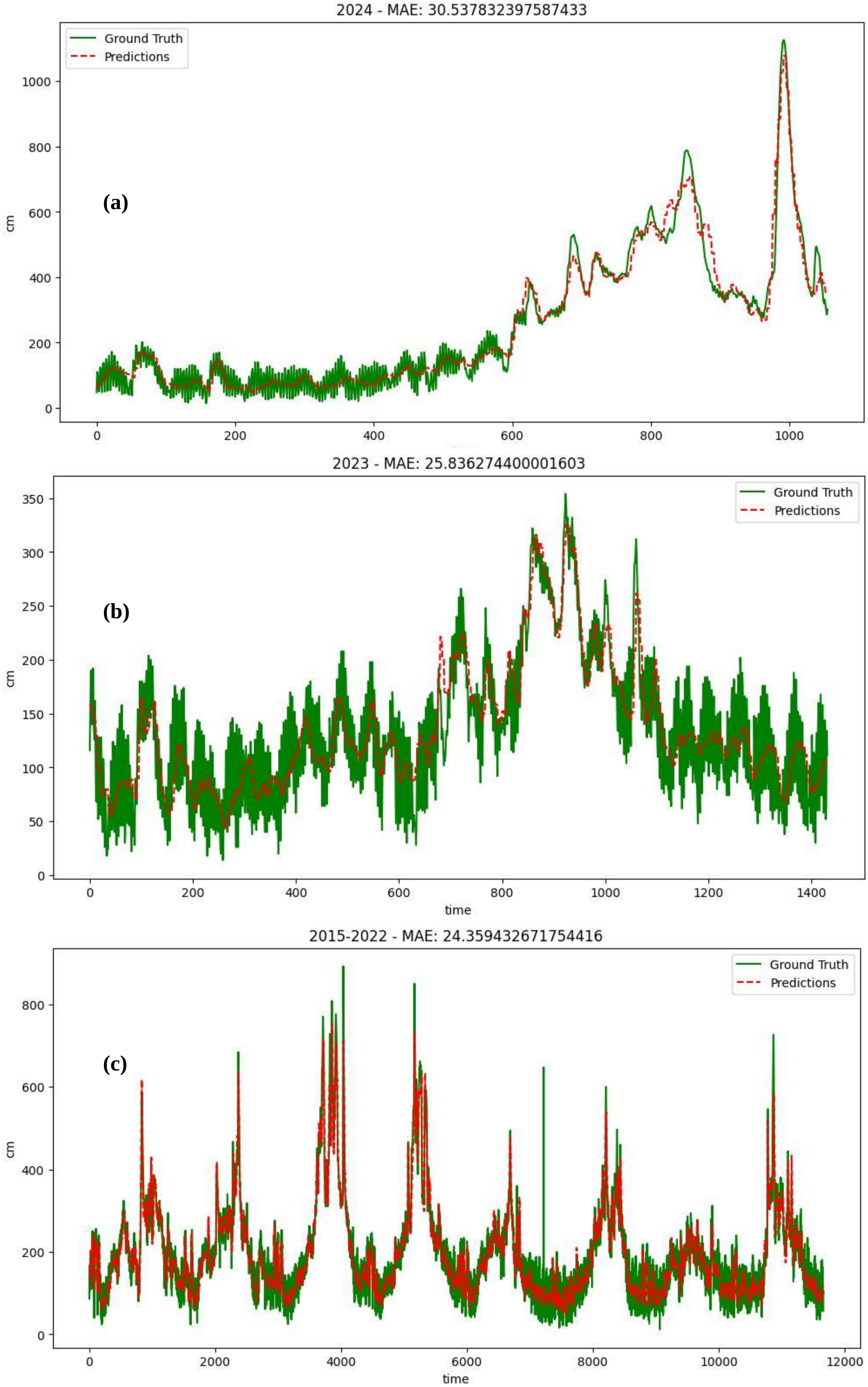


**Figure 3.** Water level at Hanoi (observed and forecasted): (a) 2024, (b) 2023, (c) 2015-2022.

A closer inspection of the charts across the dataset illustrates that the forecasted values closely track the observed values for most of the time, especially during periods of low to moderate water level fluctuations (Figure 3). The model has demonstrated a strong ability to learn historical trends and maintain high accuracy on the validation set. This is clear evidence of the Transformer architecture's effectiveness in forecasting water levels under normal conditions. Although the error is higher compared to the year 2023 (Figure 3b), the charts indicate that the model still tracks the overall trends in water levels and provides predictions that are close to the actual values. In particular, during periods of significant fluctuations, the model demonstrates a certain degree of adaptability, although there are still notable errors at some peak flood points.

In Figure 3a, the flood peak error of the September 2024 flood event is approximately -70 cm. This result can be explained by the fact that the flood in 2024 is much higher than the trained flood data in the past. Therefore, the test results are "extrapolated" values from the model's training data. However, the Transformers method still describes the flood process well and predicts the flood peak error at an acceptable level. In order to improve the quality of flood forecasts for these extreme cases, hydraulic models can be used to simulate additional data.

## 4. Conclusion

The study developed a rapid water level forecasting model for Hanoi using the Transformer-Based AI model. The model employed observational water level data from hydrological stations located along the mainstem and downstream sections of the Red and Thai Binh River basins, alongside outflow data from the primary reservoirs of Hoa Binh, Tuyen Quang, and Thac Ba. To minimize the impact of riverbed changes on water levels, only measured data collected from 2015 onward were used in the study.

The Transformer model was created using training data from 2015 to 2022, validated for 2023, and tested for 2024. The results showed that the Mean Absolute Error (MAE) was within an acceptable range, with MAE values of 24.1 cm, 26.1 cm, and 30.7 cm for the training, validation, and testing phases, respectively.

A thorough analysis of the dataset results reveals that the forecasted values align closely with the observed values for most of the time, particularly during periods of low to moderate water level fluctuations. The model has shown a strong capacity to capture historical patterns and maintain high accuracy on the validation set, highlighting the effectiveness of the Transformer architecture in forecasting water levels under normal conditions.

To improve the quality of flood forecasts for these extreme cases, hydraulic models can be used to simulate additional data.

**Author contribution statement:** study conception and design: N.L.A., T.Q.L.; Data collection: N.H.S., N.L.A.; Performed the experiments: T.N.H., N.L.A.; Analysis and interpretation of results: T.Q.L., T.N.H., N.L.A.; draft manuscript: N.L.A., T.N.H., T.Q.L.; manuscript editing: N.L.A., T.N.H.

**Acknowledgements:** This research was funded by the project "Research on the development of technology for integrating the operational rules of reservoirs system for flood forecasting in the downstream area of the Red River - Thai Binh basin", grant No. TNMT.2023.06.11.

**Competing interest statement:** The authors declare no conflict of interest.

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